

Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 1 (Individual)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
			<hr/>			
			Total score			<input type="text"/>

Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特別聲明，答案須用數字表達，並化至最簡。

1. If  $a$  is the number of real solutions of  $\frac{1}{(x+2)(x+3)} = \frac{1}{(x+1)(x+4)}$ , determine the value of  $a$ .

若  $a$  為  $\frac{1}{(x+2)(x+3)} = \frac{1}{(x+1)(x+4)}$  的實數解的數量，求  $a$  的值。

$a =$

2. If  $x$  is a real number and  $b$  is the maximum value of  $-|x-a-9|-|10-x|$ , determine the value of  $b$ .

若  $x$  為實數及  $b$  為  $-|x-a-9|-|10-x|$  的最大值，求  $b$  的值。

$b =$

3. If real numbers  $x$  and  $y$  satisfy  $4x^2 + 4y^2 + 9xy = -119b$ , determine  $c$ , the maximum value of  $xy$ .

若實數  $x$  及  $y$  滿足  $4x^2 + 4y^2 + 9xy = -119b$ ，求  $xy$  的最大值  $c$ 。

$c =$

4. If a positive real number  $x$  satisfies  $x^2 + \frac{1}{x^2} = c$ , determine the value of  $d = x^3 + \frac{1}{x^3}$ .

正實數  $x$  滿足方程  $x^2 + \frac{1}{x^2} = c$ ，求  $d = x^3 + \frac{1}{x^3}$  的值。

$d =$

**Hong Kong Mathematics Olympiad (2016/2017)**  
**Final Event 2 (Individual)**

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			Total score			<input type="text"/>

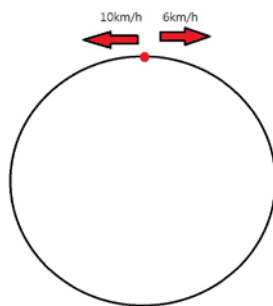
Team No.	<input type="text"/>
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除非特別聲明，答案須用數字表達，並化至最簡。

1. Two students run in opposite directions from a starting point of a 1-km circular track at speeds of 10 km/h and 6 km/h, respectively. They stop running when they meet each other at the starting point again. If  $a$  is the number of times they cross each other after they start and before they stop, determine the value of  $a$ .

兩個學生於長 1-km 的圓形跑道的起點開始分別以 10 km/h 及 6 km/h 的速率跑沿相反方向跑步。當他們於起點再相遇時便停止跑步。若  $a$  為他們開始後及停止前相互經過的次數，求  $a$  的值。



$a =$

2. There is a set of red marbles and blue marbles. When  $a$  red marbles are added to the set, the ratio of red marbles to blue marbles is 3:1. When  $a$  blue marbles are added, the ratio of red marbles to blue marbles becomes 2:1. Determine the total number of marbles,  $b$ .

袋中有若干粒紅色及藍色的彈珠。若加入  $a$  粒紅色彈珠，紅色彈珠與藍色彈珠的比例為 3:1。若加入  $a$  粒藍色彈珠，紅色彈珠與藍色彈珠的比例則為 2:1。求彈珠的總數  $b$ 。

$b =$

3. If  $c$  is the smallest difference between 1 000 000 and a square, where the square is a multiple of  $b$ , determine the value of  $c$ .

若  $c$  為 1 000 000 與一個平方數之最小的相差，其中此平方數為  $b$  的倍數，求  $c$  的值。

$c =$

4. The building of a reservoir takes  $d$  technicians, or alternatively  $y$  labours, to complete in a month, where  $d + y = c$ . If  $d$  labours are employed to build the same reservoir, the time taken is 4 times as much as the time taken when  $y$  technicians are employed. Determine the value of  $d$ .

於一個月的時間完成建築一個水庫需要  $d$  個技工或  $y$  個勞工，當中  $d + y = c$ 。若挑選  $d$  個勞工去建築一個同樣的水庫，所需要的時間較挑選  $y$  個技工的多 4 倍，求  $d$  的值。

$d =$
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Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 3 (Individual)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. If  $\{x_0, y_0, z_0\}$  is a solution to the set of simultaneous equations below, determine the value of

$$a = x_0 + y_0 + z_0.$$

$$\begin{cases} 2x - 2y + z = -15 \\ x + 2y + 2z = 18 \\ 2x - y + 2z = -5 \end{cases}$$

若  $\{x_0, y_0, z_0\}$  為以下方程組的解，求  $a = x_0 + y_0 + z_0$  的值。

$$\begin{cases} 2x - 2y + z = -15 \\ x + 2y + 2z = 18 \\ 2x - y + 2z = -5 \end{cases}$$

$a =$

2. Determine the value of  $b = \frac{(\sqrt{6+2\sqrt{a}} + \sqrt{6-2\sqrt{a}})}{2}$ .

求  $b = \frac{(\sqrt{6+2\sqrt{a}} + \sqrt{6-2\sqrt{a}})}{2}$  的值。

$b =$

3. If  $x$  is a positive integer and  $\log_{10} b^x > 3$ , determine  $c$ , the minimum value of  $x$ .

若  $x$  是正整數且  $\log_{10} b^x > 3$ ，求  $x$  的最小值  $c$ 。

$c =$

4. If  $f(x) = 2^0 + 2^1 + 2^2 + \cdots + 2^{x-2} + 2^{x-1}$ , determine the value of  $d = f(c)$ .

若  $f(x) = 2^0 + 2^1 + 2^2 + \cdots + 2^{x-2} + 2^{x-1}$ ，求  $d = f(c)$  的值。

$d =$
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Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 4 (Individual)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. If  $a$  is a positive integer, determine the greatest value of  $a$  such that  $ax^2 - (a-3)x + (a-2) = 0$  has real roots.

若  $a$  為正整數，求  $a$  的最大值使得  $ax^2 - (a-3)x + (a-2) = 0$  有實根。

$a =$

2. If  $x$  and  $y$  are real numbers with  $1 < y < x$  and  $\log_x y + 3\log_y x = \frac{13}{a}$ , determine the value of

$$b = \frac{x+y^4}{x^2+y^2}.$$

若  $x$  及  $y$  為實數且  $1 < y < x$  及  $\log_x y + 3\log_y x = \frac{13}{a}$ ，求  $b = \frac{x+y^4}{x^2+y^2}$  的值。

$b =$

3. A bag contains  $b+2$  red balls,  $b+3$  white balls and  $b+4$  blue balls. Three balls are randomly drawn from the bag without replacement. Determine the value of the probability,  $c$ , that the 3 balls are of the same colour.

一個袋中有紅球  $b+2$  個，白球  $b+3$  個及藍球  $b+4$  個，從袋中隨機抽出 3 個球並不重新放進袋中。求三個抽出的球都是相同顏色的概率  $c$  的值。

$c =$

4. If  $\cos 2\theta = c$ , determine the value of  $d = \sin^4 \theta + \cos^4 \theta$ .

若  $\cos 2\theta = c$ ，求  $d = \sin^4 \theta + \cos^4 \theta$  的值。

$d =$